Chapter 10 - Strains
Reading: Chapter 10 - Pages 357—362

10-1
Introduction

Structural materials deform under the action of forces.

There are three kinds of deformation:

Elongation	an increase in length
Contraction	decrease in length
Angular Distortion	a change in shape

Deformation (either elongation or contraction) per unit length is called *linear strain*.

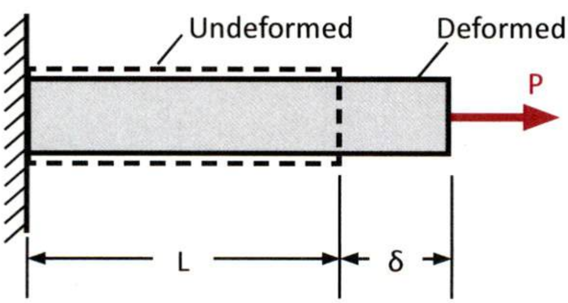
10-2
Linear Strain

Linear strain of a deformed body is defined as the ratio of the change in length of the body due to the deformation to its original length in the direction of the force.

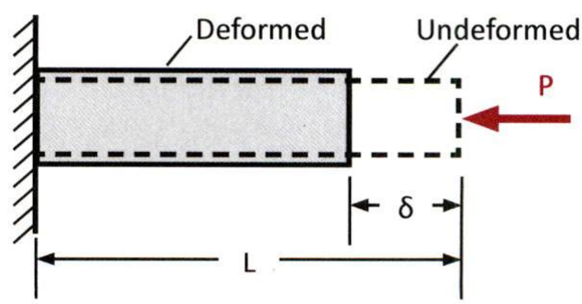
Linear Strain

$$\varepsilon = \frac{\delta}{L}$$

where ε = the linear strain
δ = total axial deformation (elongation or contraction)
L = the original length of the member



Elongation



Contraction

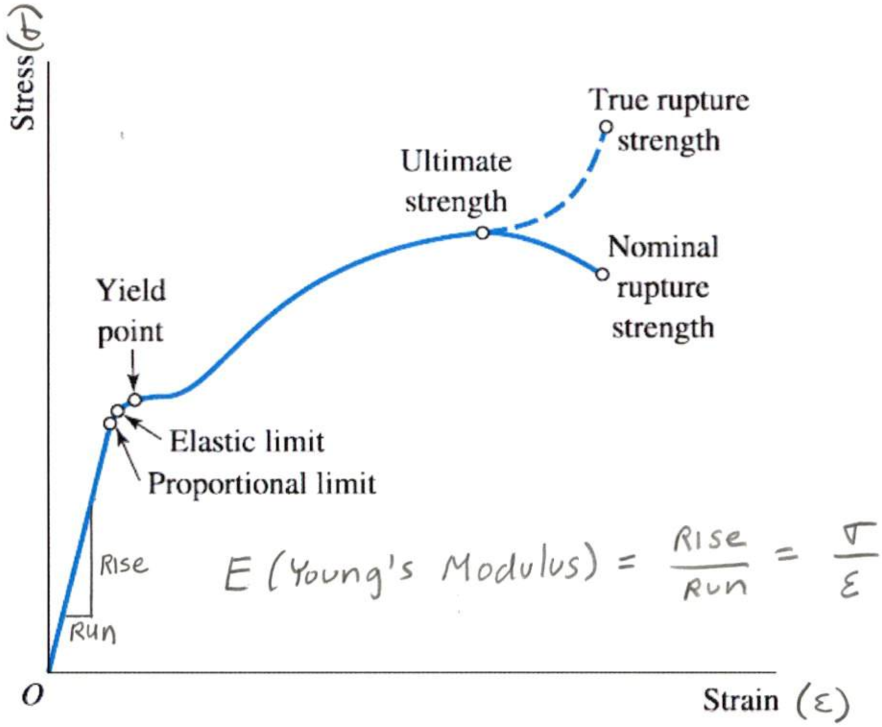
Linear strain may be tensile or compressive strain according to an increase in length or decrease in length of the body.

The convention used in this book is deformation caused by tensile forces are usually considered positive; those caused by compressive force are considered negative.

10-3
Hooke's Law

For most engineering materials a linear relationship exists between stress and strain; that is, up to a certain limiting value of stress, the stress is proportional to the strain.

The linear relationship between stress and strain is known as Hooke's Law.



Hooke's Law

$$\frac{\sigma}{\varepsilon} = E$$

or

$$\sigma = E\varepsilon$$

where E is the constant of proportionality between stress and strain and is called the *modulus of elasticity*.

Example

10-1 A 10-ft steel bar is subjected to a tensile stress of 20 ksi. Determine (a) the linear strain and (b) the total deformation of the bar. The modulus of elasticity of the steel is 30 x 10³ ksi.

Solution.

(a) From Hooke's Law, $\epsilon = \frac{\sigma}{E} = \frac{20 \text{ ksi}}{30 \times 10^3 \text{ ksi}} = 0.000667$
$= 6.7 \times 10^{-4}$

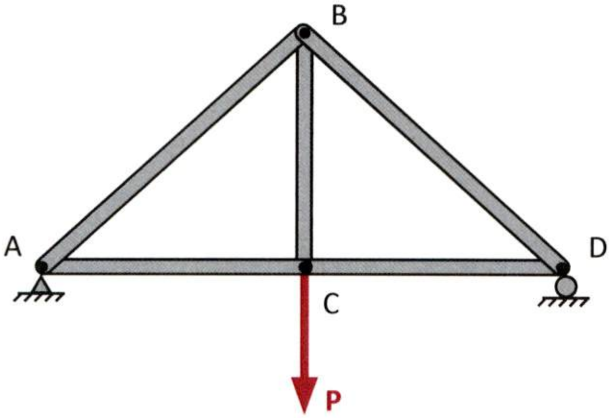
(b) $\delta = \varepsilon L = 0.000667\,(10 \text{ ft}) = 0.0067 \text{ ft}$ (axial deformation)

$= 0.0067 \text{ ft} \times \frac{12 \text{ in}}{\text{ft}}$

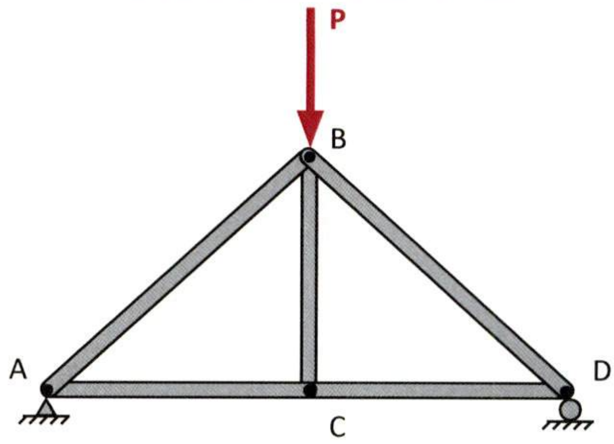
$= 0.08 \text{ in}$

Axial Deformation

An axially loaded member elongates under a tensile load and contracts under a compressive load.



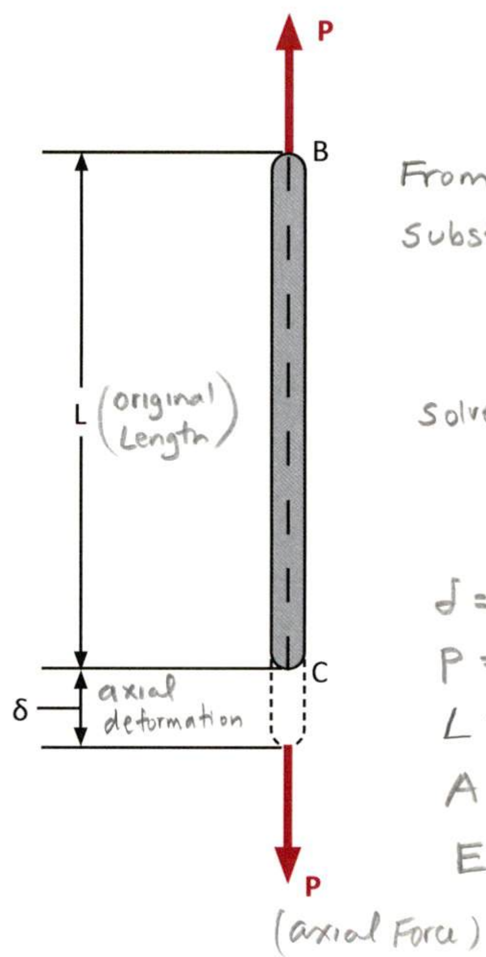
Member BC is axially loaded in Tension (T)



Member BC is axially loaded in Compression (C)

If the normal stress in an axially loaded member is within the proportional limit of the material, Hooke's Law applies and the axial deformation of the member can be computed.

Member BC has a cross-sectional area A and is subjected to an axial tensile force P.



Stress

$$\sigma = \frac{P}{A}$$

Strain

$$\varepsilon = \frac{\delta}{L}$$

From Hooke's Law, $\sigma = E\varepsilon$

subst. in σ & ε

$$\frac{P}{A} = E\frac{\delta}{L}$$

solve for δ

$$\boxed{\delta = \frac{PL}{AE}}$$

δ = the total axial deformation

P = the applied axial Load

L = the original length of the member

A = the cross-sectional area of the member

E = the modulus of elasticity of the material of the member

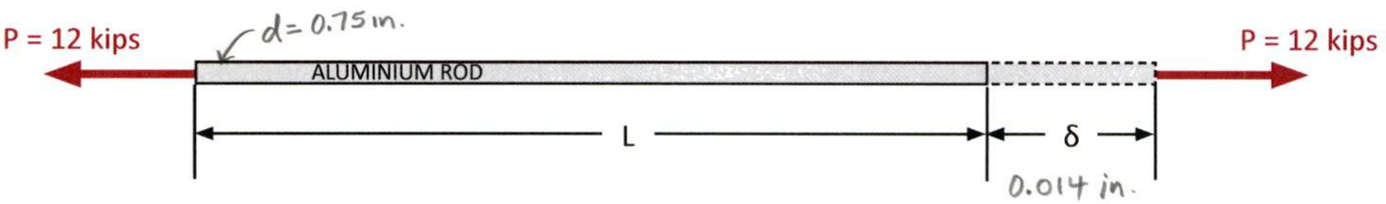
Important Points about the Axial Deformation Equation

$$\delta = \frac{PL}{AE}$$

- It is valid only when the normal stress in the member does not exceed the proportional limit.
- For most structural materials, the modulus of elasticity for tension and for compression are the same.
- Deformation caused by tensile forces are usually considered positive; those caused by compressive force are considered negative.
- If a member is subjected to several axial forces, the axial deformation of each segment must be calculated. The total axial deformation of the member is the algebraic sum of the axial deformation of all the segments.

Example

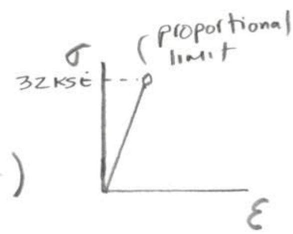
A aluminium rod has a diameter of ¾ in. and is subjected to a tensile axial load of 12 kips. The rod has an elongation of δ = 0.014 in. along the longitudinal direction. Determine the original length of the rod. The modulus of elasticity of the aluminium is E = 10.3 x 10³ ksi and the proportional limit is 32 ksi.



Solution.

The tensile stress in the rod is

$$\sigma = \frac{P}{A} = \frac{12 \text{ kips}}{\frac{\pi (0.75 \text{in})^2}{4}} = 27 \text{ ksi}$$



Since 27 ksi < 32 ksi (the proportional limit)

$\Rightarrow$ Hooke's Law Applies

$$\delta = \frac{PL}{AE} \Rightarrow L = \frac{\delta AE}{P}$$

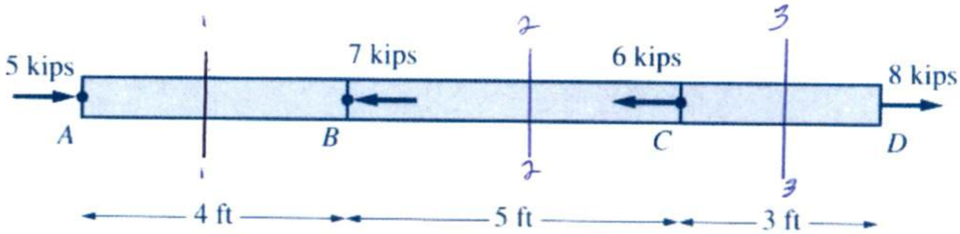
and

$$L = \frac{0.014 \text{in} \left(\frac{\pi \, 0.75 \text{in}^2}{4}\right)(10.3 \times 10^3 \text{ ksi})}{12 \text{ kips}}$$

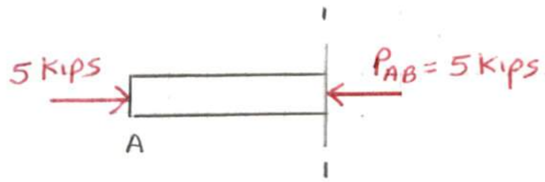
$$= 5.3 \text{ in.}$$ (original length of the rod)

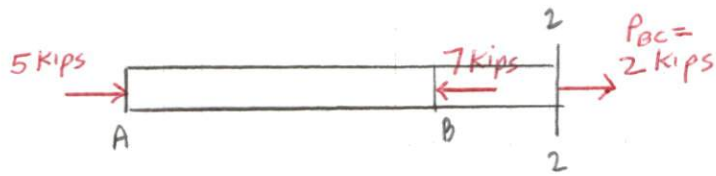
Example 10-2

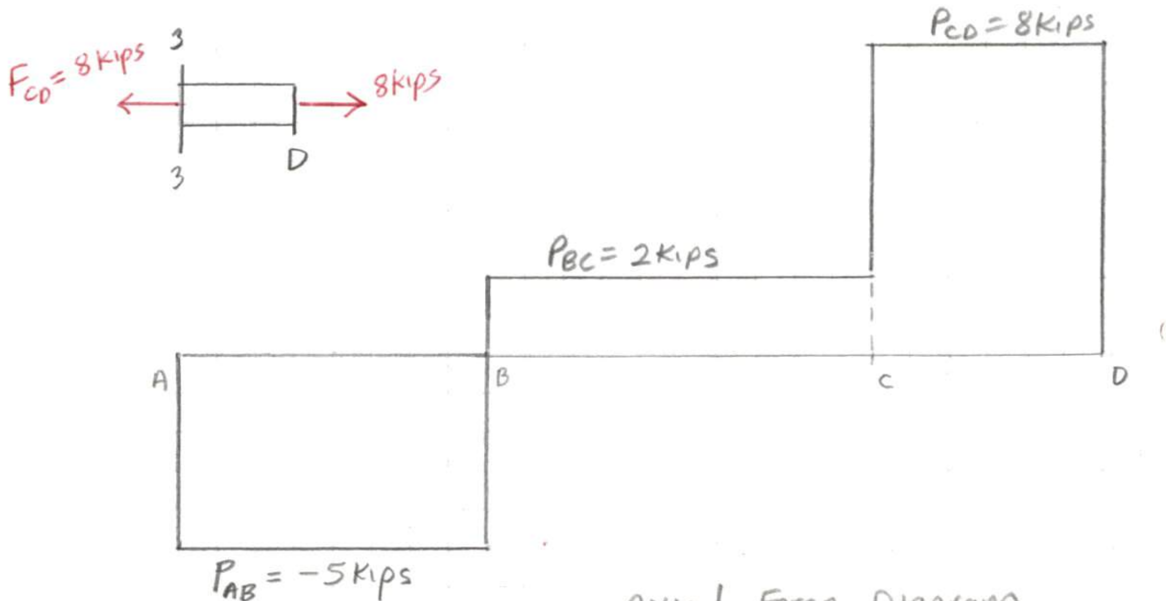
Determine the total axial deformation of the steel bar between sections A and D. The bar has a cross-section ½ in. X 1 in. and is subjected to the axial loads shown. The modulus of elasticity of steel is E = 30 X 10³ ksi and the proportional limit is 34 ksi.



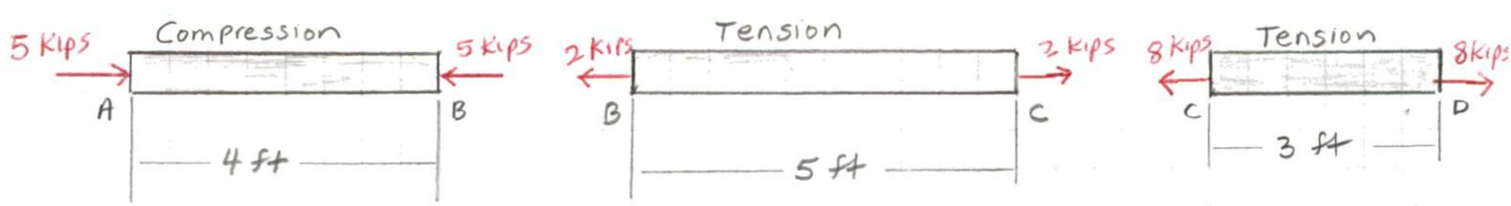
Solution.







Axial Force Diagram



The maximum load occurs in segment CD. $\sigma = \frac{P}{A} = \frac{8 \text{ kips}}{0.5 \text{ in}^2} = 16 \text{ ksi}$

since 16 ksi < 34 ksi (proportional limit)

$\Rightarrow$ Hooke's Law Applies

The axial deformation of each segment is

$$\delta_{AB} = \frac{(-5\text{kips})(4\text{ft})}{(0.5\text{in} \times 1\text{in})(30\,000\ \text{kips}/\text{in}^2)} = -0.00133\ \text{ft}$$

$$\delta_{BC} = \frac{(2\text{kips})(5\text{ft})}{15000\ \text{kips}} = +0.00066\ \text{ft}$$

$$\delta_{CD} = \frac{(8\text{kips})(3\text{ft})}{15000\ \text{kips}} = +0.00160\ \text{ft}$$

$$\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD}$$

$$= -0.00133\ \text{ft} + 0.00066\ \text{ft} + 0.00160\ \text{ft}$$

$$= 0.00093\ \text{ft}$$

$$= \underline{\underline{0.0112\ \text{in.}}}\ \text{(elongation)}$$